

Nuclear suppression of heavy quark production at forward rapidities in relativistic heavy ion collisions

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Abstract. We calculate nuclear suppression R_{AA} of heavy quarks produced from the initial fusion of partons in nucleus-nucleus collisions at RHIC and LHC energies. We take the shadowing as well as the energy loss suffered by them while passing through Quark Gluon Plasma into account. We obtain results for charm and bottom quarks at several rapidities using different mechanisms for energy loss, to see if we can distinguish between them.

1. Introduction

The heavy ion collision experiments at RHIC and LHC are designed with a hope to explore the existence of a new form of matter known as Quark Gluon Plasma (QGP) and to explore its properties. The estimation for the energy density [1, 2, 3, 4] attained in these collisions using the Bjorken formula [5] is well beyond the energy densities where QGP is expected to be formed. The temperatures reached at RHIC, as revealed from several studies (see e.g., [6, 7, 8] for a compilation) are also much larger than the values provided by Lattice QCD calculations for the critical temperature for a transition to QGP [9]. Strong confirmation of the formation of the QGP is given by observation of a large elliptic flow [10], jet-quenching [11], and the recombination of partons as the mechanism of production of hadrons at intermediate transverse momenta [12]. Still higher temperatures are likely to be reached at LHC.

Heavy quarks are produced from the initial fusion of gluons ($gg \rightarrow Q\bar{Q}$) or light quarks ($q\bar{q} \rightarrow Q\bar{Q}$). This pair would be produced at $\tau \approx 1/2M_Q \ll 0.1$ fm/c. Their large mass ensures that their production can be treated using pQCD and that it is nearly negligible at later times. These will traverse the QGP, colliding with quarks and gluons and radiating gluons before appearing as charm or bottom mesons or baryons. Thus the final spectra for these hadrons would contain information about the energy loss suffered by the heavy quarks. The unique importance of heavy quarks as probes of QGP lies in their large mass. This leads to a considerably reduced production of heavy quarks in comparison to light quarks and gluons which are produced copiously. The strong interaction during the collision conserves flavour. Even at the LHC, the reverse

process of annihilation of heavy quarks ($Q\bar{Q} \rightarrow gg$, etc.) can be safely ignored. Thus heavy quarks and in turn charm and bottom hadrons will stand out in the back-ground of a multitude of light hadrons, and one can in principle track them.

Along with other reasons, an interest in the study of energy loss of heavy quarks was triggered by the large back-ground that correlated charm or bottom decay provides [13, 14] to the thermal dileptons which have been considered a signature of the formation of QGP for a long time [15, 16, 17, 18, 19]. It was pointed out by Shuryak [20], Lin *et al.* [21, 22], Kampfer *et al.* [23], and Mustafa *et al.* [24, 25] that the correlated charm and bottom decay could be suppressed if the energy loss suffered by heavy quarks before they form D or B mesons was accounted for. Since then several attempts have been made to estimate the energy loss of heavy quarks as they proceed through QGP. Of course the possibility to identify the vertex of D or B meson decay will further enrich this study.

These results have been put to a rigorous test by the measurement of single electrons from heavy ion collisions at RHIC which show a clear evidence for the loss of energy by the heavy quarks [26]. Their possible thermalization is also indicated by the elliptic flow that they show [27].

As indicated earlier, the temperature likely to be reached at LHC in collision of heavy nuclei could be even larger and thus this energy loss will play a more significant role. The opening of a much wider window in rapidity at LHC is also likely to provide widely differing media at different rapidities through which the heavy quarks would propagate.

Thus a valuable test of various theories for energy loss suffered by heavy quarks can be performed by studying it at RHIC and LHC and at different rapidities.

We study these effects in terms of nuclear modification factor R_{AA} for heavy quarks. In these initial studies we calculate the average energy loss suffered by them as they pass through the QGP and compare the resulting p_T distribution with the same for proton-proton (pp) collisions to get R_{AA} . Since the mass of charm or bottom quarks is quite large, the p_T distribution of these quarks will closely reflect the p_T distribution of D or B mesons.

We employ a local fluid approximation [13, 14, 28] in order to picture the medium at larger rapidities. We shall come back to this later.

The paper is organized as follows. As we need to compare the spectra of the heavy quarks from relativistic heavy ion collisions with those for pp collisions, as a first step we study the heavy quark production in LO pQCD and compare our results with a NLO pQCD calculation. We find that single quark distribution calculated using LO pQCD supplemented with a K-factor adequately reproduces the NLO results as well as the available experimental data. Next we estimate the average energy loss suffered by heavy quarks of a given energy using various mechanisms discussed in the literature.

Finally we perform a Monte Carlo calculation to obtain the average change in the transverse momentum spectra of heavy quarks for nucleus-nucleus collisions and get R_{AA} as a function of p_T for different rapidities. We add that this work is not intended

as a complete review and the readers may see Ref. [29, 30], for other treatments.

More detailed calculations where the consequences of energy loss of heavy quarks on the correlated charm or bottom decay and modification of the back-to-back correlation of heavy quarks are discussed, will be published shortly.

2. Heavy quark production in pp collisions

At lowest order in pQCD, heavy quarks in pp collisions are produced by fusion of gluons ($gg \rightarrow Q\bar{Q}$) or light quarks ($q\bar{q} \rightarrow Q\bar{Q}$) [31]. The so-called flavour excitation process ($qQ \rightarrow qQ$ and $gQ \rightarrow gQ$) is now known to be suppressed when the NLO processes are taken into account [32, 33, 34]. In addition, Brodsky *et al.* [35, 36] have shown that the total contribution of intrinsic charm in the midrapidity region is small even though most of the heavy quarks are produced in this region.

The cross-section for the production of heavy quarks from pp collisions at lowest order is given by [31, 37]:

$$\frac{d\sigma}{dy_1 dy_2 dp_T} = 2x_1 x_2 p_T \sum_{ij} [f_i^{(1)}(x_1, Q^2) f_j^{(2)}(x_2, Q^2) \hat{\sigma}_{ij}(\hat{s}, \hat{t}, \hat{u}) + f_j^{(1)}(x_1, Q^2) f_i^{(2)}(x_2, Q^2) \hat{\sigma}_{ij}(\hat{s}, \hat{t}, \hat{u})] / (1 + \delta_{ij}). \quad (1)$$

In the above equation, i and j are the interacting partons, $f_i^{(1)}$ and $f_j^{(2)}$ are the partonic structure functions and x_1 and x_2 are the fractional momenta of the interacting hadrons carried by the partons i and j . The relation between p_T and fractional momentum x_1 or x_2 through their respective rapidities can be written as

$$x_1 = \frac{m_T}{\sqrt{s}} (e^{y_1} + e^{y_2}), \quad x_2 = \frac{m_T}{\sqrt{s}} (e^{-y_1} + e^{-y_2}), \quad (2)$$

where m_T is the transverse mass, $\sqrt{M^2 + p_T^2}$, of the produced heavy quark. The function $\hat{\sigma} = d\sigma/dt$, the short range subprocess for the heavy quark production is defined as:

$$\frac{d\sigma}{dt} = \frac{1}{16\pi\hat{s}^2} |\mathcal{M}|^2. \quad (3)$$

$|\mathcal{M}|^2$ for the heavy quark production processes $gg \rightarrow Q\bar{Q}$ and $q\bar{q} \rightarrow Q\bar{Q}$ are expressed through the mass of the heavy quark and Mandelstam variables \hat{s} , \hat{t} , and \hat{u} as

$$\begin{aligned} \left| \mathcal{M} \right|_{(gg \rightarrow Q\bar{Q})}^2 &= \pi^2 \alpha_s^2 \left[\frac{12}{\hat{s}^2} (M^2 - \hat{t}) (M^2 - \hat{u}) \right. \\ &\quad + \frac{8}{3} \frac{(M^2 - \hat{t}) (M^2 - \hat{u}) - 2M^2 (M^2 + \hat{t})}{(M^2 - \hat{t})^2} \\ &\quad \left. + \frac{8}{3} \frac{(M^2 - \hat{t}) (M^2 - \hat{u}) - 2M^2 (M^2 + \hat{u})}{(M^2 - \hat{u})^2} \right] \end{aligned}$$

$$\begin{aligned}
& - \frac{2M^2 (\hat{s} - 4M^2)}{3 (M^2 - \hat{t}) (M^2 - \hat{u})} \\
& - 6 \frac{(M^2 - \hat{t}) (M^2 - \hat{u}) + M^2 (\hat{u} - \hat{t})}{\hat{s} (M^2 - \hat{t})} \\
& - 6 \frac{(M^2 - \hat{t}) (M^2 - \hat{u}) + M^2 (\hat{t} - \hat{u})}{\hat{s} (M^2 - \hat{u})} \Big] \quad (4)
\end{aligned}$$

and

$$\left| \mathcal{M} \right|_{(q\bar{q} \rightarrow Q\bar{Q})}^2 = \frac{64\pi^2\alpha_s^2}{9} \left[\frac{(M^2 - \hat{t})^2 + (M^2 - \hat{u})^2 + 2M^2\hat{s}}{\hat{s}^2} \right]. \quad (5)$$

The running coupling constant α_s at lowest order is

$$\alpha_s = \frac{12\pi}{(33 - 2N_f) \ln(Q^2/\Lambda^2)}, \quad (6)$$

where $N_f = 3$ is the number of active flavours and $\Lambda = \Lambda_{\text{QCD}}$. We use the factorization and renormalization scales as $Q^2 = m_T^2$. We refer the readers to Vogt *et al.* [38] for results on variations of these scales. We also carry out the calculation of differential cross section for heavy quarks in pp collision at NLO in pQCD using the treatment developed by Mangano, Nason, and Ridolfi (MNR-NLO) [39]. All the calculations are carried out by neglecting the intrinsic transverse momentum of the partons.

The effect of nuclear shadowing in high energy nucleus-nucleus collisions is well known [40, 41, 42, 43]. With the increase of the mass number of the nucleus and increasing contribution of terms having small x , the effect becomes more pronounced. We introduce the shadowing effect in our calculations by using EKS 98 parameterization [44] for nucleon structure functions. We take CTEQ4M [45] structure function set for nucleons.

We shall see that x dependence of the shadowing function introduces interesting structures in the nuclear modification factor as a function of p_T , y , and the incident energy, because of the large mass of the quarks.

In Fig. 1 we compare our results for heavy quark p_T distribution obtained using lowest order pQCD for pp collision with the results from NLO-MNR calculation at midrapidity for charm and bottom quarks at RHIC and LHC energies. These comparisons suggest a K factor of ≈ 1.5 - 3 for our lowest order calculations for agreement with NLO results.

In Fig. 2 we compare the bottom quark production cross-section obtained using lowest order pQCD in pp collision at 630 and 1800 GeV energies with UA1, CDF and $D\bar{O}$ data [46, 47]. We find a good description of these data using lowest order pQCD with a K-factor.

We calculate the total cross-section for charm quark production at lowest order for the process $pp \rightarrow c\bar{c}$ as a function of \sqrt{s} considering the charm quark mass as 1.2 GeV and 1.6 GeV. We have also included results for $M_c(3 \text{ GeV}) = 0.986 \text{ GeV}$ [48], suggested

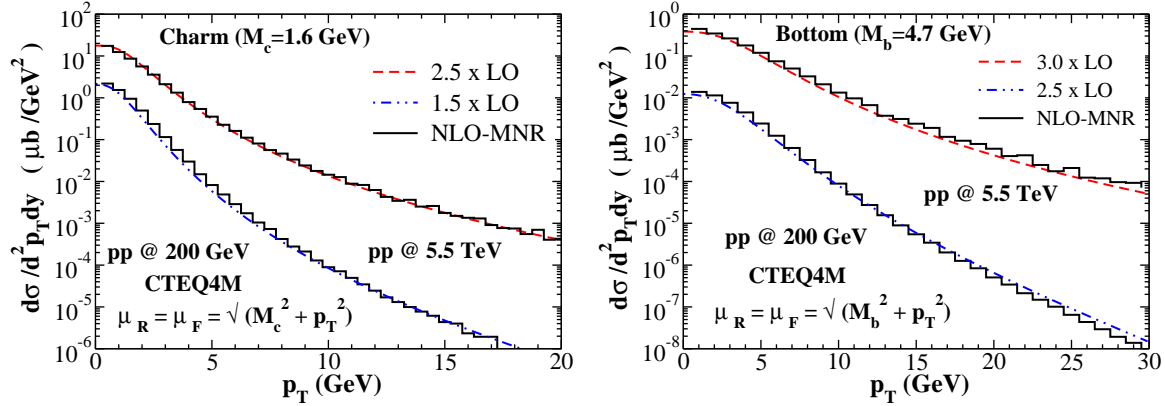


Figure 1. [Left panel] Comparison of our lowest order pQCD results with the NLO-MNR calculation for charm quark ($M_c = 1.6$ GeV) at midrapidity. [Right panel] Same for bottom quark ($M_b = 4.7$ GeV) at midrapidity.

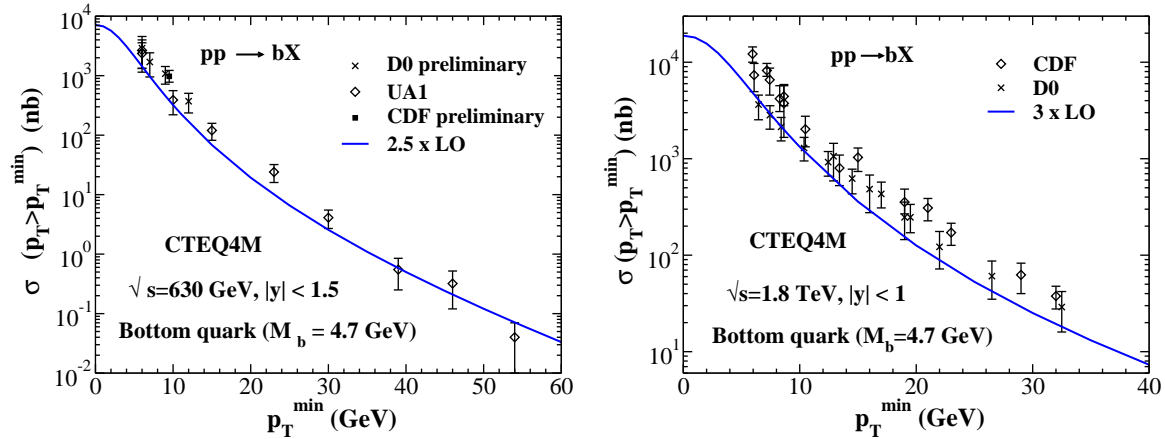


Figure 2. Comparison of our result for $\sigma(p_T > p_T^{\min})$ for production of bottom quarks with experimental data.

recently. Though the last value is not at the pole $Q = M_c$, it may serve as the lower limit to the mass of the charm quark. In the left panel of Fig. 3 we compare these results and the results from NLO-MNR calculation with the experimental data points [47, 49, 50]. Here also our lowest order pQCD calculation show a good agreement with experimental data points for $M_c = 1.2$ GeV. In the right panel of Fig. 3 we compare our results with the NLO-MNR calculations up to $\sqrt{s} = 15000$ GeV.

We also calculate the total cross-section for bottom quark production for $pp \rightarrow b\bar{b}$ as a function of \sqrt{s} considering the bottom quark mass as 4.7 GeV and 4.163 GeV [48]. In Fig. 4 we compare these results and the results obtained from NLO-MNR calculation with the experimental data points [51]. This comparison is quite impressive as at $M_b = 4.7$ GeV our lowest order result accurately reproduce the result of NLO-MNR calculation with $K = 2.5$.

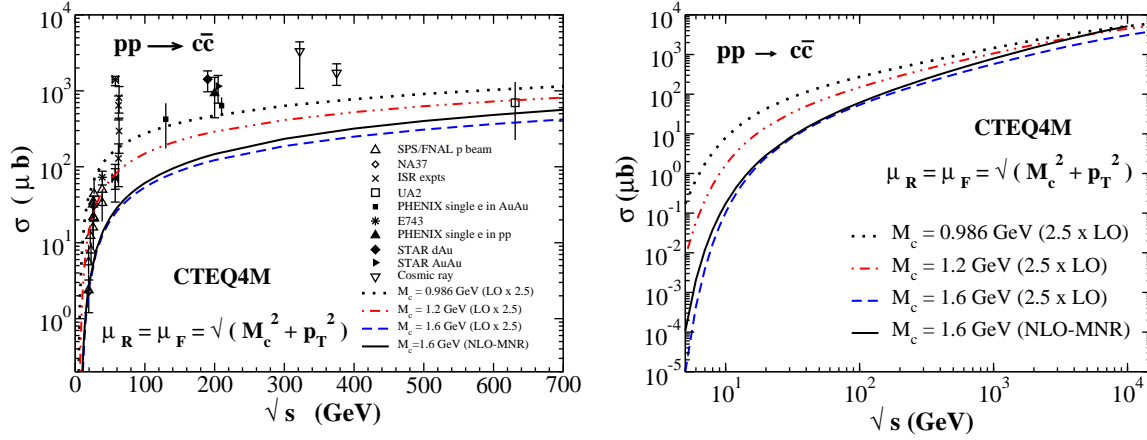


Figure 3. Total cross-section for $pp \rightarrow c\bar{c}$ compared with experimental data at varying \sqrt{s} .

Thus we see that the p_T distribution and production cross-section for the heavy quarks calculated in lowest order pQCD and supplemented with a K-factor ≈ 2.5 reproduces the results at NLO for pp collisions. In view of this we feel that these distribution would be adequate for calculating R_{AA} .

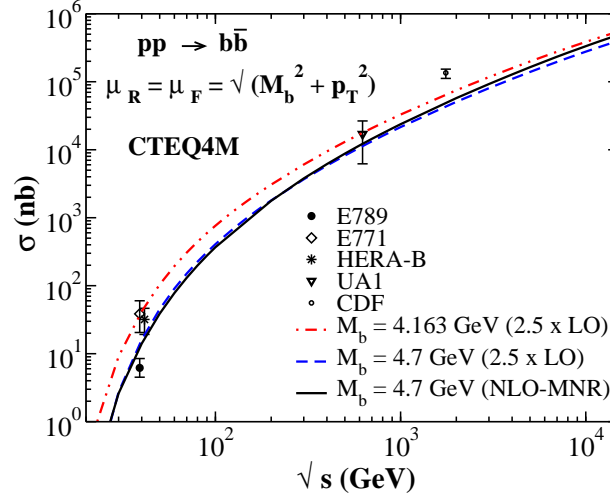


Figure 4. Total cross-section for $pp \rightarrow b\bar{b}$ compared with experimental data varying \sqrt{s} .

3. The Initial conditions and the Evolution of the Plasma

The heavy quarks produced at the initial stage pass through the QGP, where they lose energy by colliding with quarks and gluons and also by radiating gluons. The energy loss will depend upon the path-length of the heavy quarks in the plasma, the temperature evolution of the plasma, and the energy and mass of the heavy quarks.

In order to proceed we make several simplifying assumptions. It is expected that the heavy quarks will loose most of their energy when the temperature is still large, i.e. during the earliest times after the formation of QGP. During these early times, we can neglect the transverse expansion of the plasma. We assume a Gaussian rapidity density distribution for the particles produced and further assume that the initial rapidity distribution of the quarks and gluons follows this distribution closely. This would correspond to an isentropic expansion of the plasma at all rapidities, and should be sufficient for our initial studies.

Thus we assume the rapidity distribution of the density of gluons as [13, 14]:

$$\frac{dN_g}{dy} = \left(\frac{dN_g}{dy} \right)_0 \exp \left(-y^2/2\sigma^2 \right). \quad (7)$$

We take $(\frac{dN_g}{dy})_0 \approx 900$ and $\sigma = 3$ for Au+Au collisions at RHIC [52] and ≈ 3300 and $\sigma = 4$ for Pb+Pb collisions at LHC [53].

The Bjorken cooling is then assumed to work locally at different rapidities, and we consider the passage of a heavy quark having rapidity y in a fluid having an identical fluid rapidity. This approximation, which corresponds to assuming a boost-invariant expansion along with a local fluid approximation, has been used earlier in literature [13, 14, 28]. A more complete study would use a $(3+1)$ dimensional hydrodynamics [54, 55], which we plan to use in future publications.

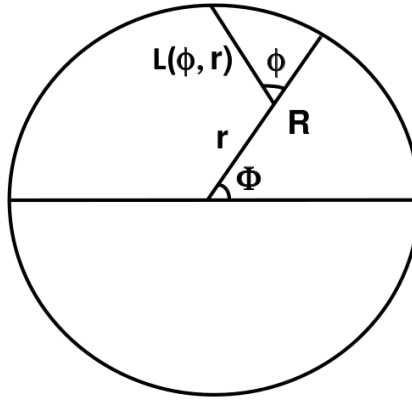


Figure 5. The distance, L , covered by a heavy quark while passing through the QGP. For central collisions the results for $\langle L \rangle$ will not depend on Φ .

We consider a heavy quark produced in a central collision, at the point (r, Φ) , and moving at an angle ϕ with respect to \hat{r} in the transverse plane. In general the distance covered by the heavy quark before it exists the QGP, will vary from 0 to $2R$, where R is the radius of the colliding nuclei. The distance covered by the heavy quark in the plasma, L , is given by [56]:

$$L(\phi, r) = \sqrt{R^2 - r^2 \sin^2 \phi} - r \cos \phi. \quad (8)$$

We can estimate the average distance travelled by the heavy quarks in the plasma as:

$$\langle L \rangle = \frac{\int_0^R r dr \int_0^{2\pi} L(\phi, r) T_{AA}(r, b=0) d\phi}{\int_0^R r dr \int_0^{2\pi} T_{AA}(r, b=0) d\phi}. \quad (9)$$

In the above the nuclear overlap function $T_{AA}(r, b=0)$ provides the probability of production of heavy quarks in hard binary collisions. We find that $\langle L \rangle$ is 5.78 fm for Au+Au collisions at RHIC and 6.14 fm for Pb+Pb collisions at LHC, and is about 20% smaller than the radii of the colliding nuclei, as the appearance of the nuclear overlap function gives a larger weight to the points having smaller r .

As the heavy quarks loose most of their energy in interaction with gluons, it is enough to consider only the distribution of gluons. Their density at the time τ can be written as [57]:

$$\rho(\tau) = \frac{1}{\pi R^2 \tau} \frac{dN_g}{dy}. \quad (10)$$

The corresponding temperature [57], assuming a chemically equilibrated plasma is

$$T(\tau) = \left(\frac{\pi^2}{1.202} \frac{\rho(\tau)}{(9 N_f + 16)} \right)^{\frac{1}{3}}. \quad (11)$$

The rapidity dependence of the temperature of the plasma at a typical $\tau = \langle L \rangle / 2$ at RHIC and LHC is given in Fig. 6.

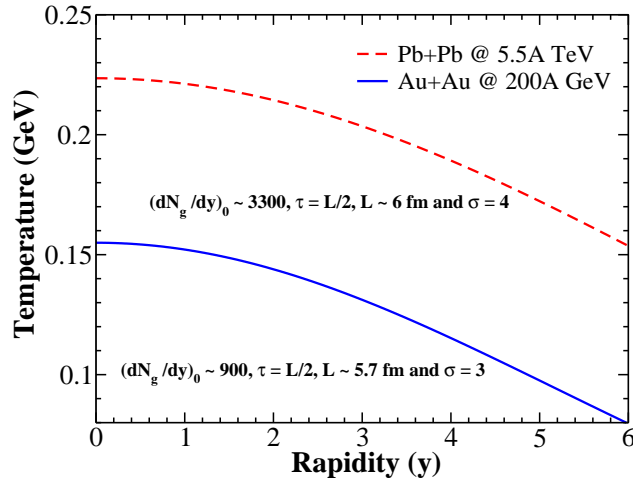


Figure 6. Variation of temperature of QGP with rapidity y at a typical τ .

Assuming that the QGP is formed at $\tau_0 = 0.2$ fm/c, we estimate the T_0 at $y=0$ for RHIC as 377 MeV and LHC as 555 MeV. More detailed studies do suggest a larger formation time of ≈ 0.5 fm/c, which will correspondingly reduce the initial temperatures. This will not affect our results as we approximate the expanding and cooling plasma,

with one at a temperature determined at $\tau = L_{\text{eff}}/2$, which is much larger (see the following discussion). Further assuming, Bjorken's cooling law, $T^3 \tau = \text{constant}$, this provides that the plasma would cool down to the transition or critical temperature $T_c \approx 160$ MeV by $\tau_c \approx 2.6$ fm/c at RHIC and 8 fm/c at LHC. One can easily calculate the corresponding values at larger rapidities.

Considering the velocity of the quark as $v_T = p_T/m_T$, it would take a time $\tau_L = \langle L \rangle / v_T$ to cross the plasma. If $\tau_c \geq \tau_L$ the heavy quark would be inside QGP during the entire period, τ_0 to τ_L . However, if $\tau_c < \tau_L$, only while covering the distance $v_T \times \tau_c$, would the heavy quark be in the QGP phase. We further approximate the expanding and cooling plasma with one at a temperature of T at $\tau = \langle L \rangle_{\text{eff}}/2$, where $\langle L \rangle_{\text{eff}} = \min [\langle L \rangle, v_T \times \tau_c]$. This procedure has been used frequently [57].

4. Mechanisms for Energy Loss

Next we discuss the energy loss mechanisms that we have included. As mentioned above, repeatedly, the heavy quarks loose energy both by collisions as well as radiation of gluons. A number of formalisms have been proposed for the collisional as well as the radiative energy loss of heavy quarks in the literature. We shall consider the following treatments for the collisional energy loss.

Bjorken [58] has considered the collisional energy loss of light quarks as analogous to the energy loss of a charged particle passing through a medium and losing energy by ionizing the medium. His expression for massless quarks was adapted by Braaten and Thoma to the case of heavy quarks [see Eq. A.1]. We shall continue to label this mechanism as Bjorken for clarity. Braaten and Thoma (BT) [59, 25] also modified the expression for the energy loss suffered by muons while traversing QED plasma, to obtain the collisional energy loss of a heavy quark as it passes through the QGP [see Eqs. A.2 and A.3 in the Appendix A]. These results are valid for collisions where the momentum transfer $q \ll E$, where E is the energy of the heavy quark. Peigne and Peshier (PP) [60] have improved this treatment by including the u-channel, which becomes important for large energies [see Eq. A.4 in the Appendix A].

For the calculation of radiative energy loss, we consider the treatment of Djordjevic, Gyulassy, Levai, and Vitev (DGLV) [61, 57] using opacity expansion, the treatment of Armesto, Salgado, and Wiedemann (ASW) [62] using path integral formalism for medium-induced gluon radiations off massive quarks, and the treatment of Xiang, Ding, Zhou, and Rohrich (XDZR) [63] using light cone path integral approach. Detailed expressions for these formalisms are given in the Appendix B.

5. Results for Energy Loss

We compare the results for transverse energy loss for a heavy quark using these different energy loss treatments for several rapidities. We plot the transverse energy loss of charm and bottom quarks, ΔE_T as a function of transverse energy E_T ($\sqrt{p_T^2 + M^2}$) in Figs. 7

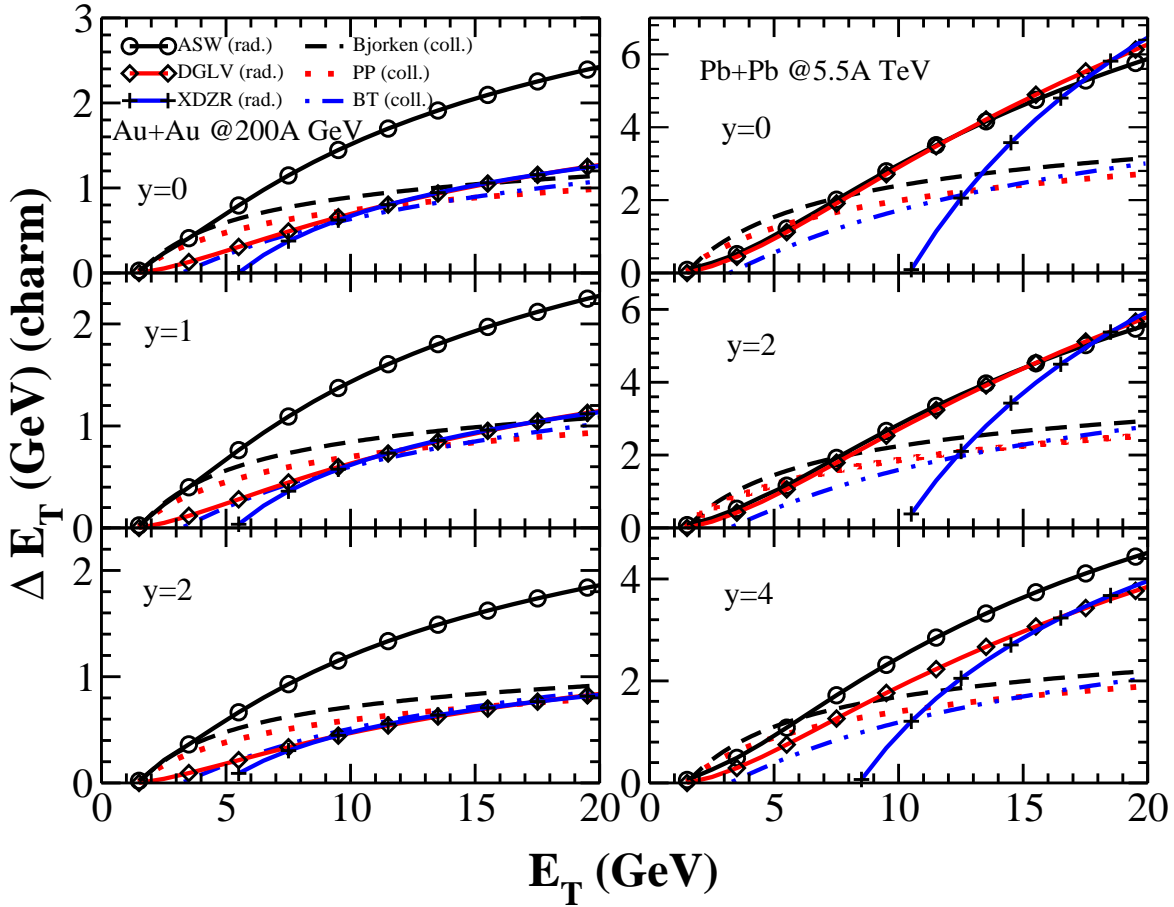


Figure 7. Collisional (dotted lines) and radiative (solid lines) energy loss suffered by a charm quark while passing through the QGP

and 8, at RHIC and LHC energies.

Several interesting features emerge. We see that the collisional energy loss for charm quarks at RHIC and LHC energies is only marginally dependent on the rapidity and the BT formalism gives largest energy loss, as expected. In our treatment, change of rapidity implies a change in the temperature of the plasma. Thus these results suggest a weaker dependence on the temperature and the average path length for the energy loss suffered by charm quarks due to collisions.

The radiative energy loss, on the other hand, shows a much more complex behaviour and is quite different for the different formalisms under consideration. We note that the ASW formalism for radiative energy loss gives largest degradation in the energy at all rapidities (except for $E_T < 5$ GeV at LHC, where it is comparable to the collisional energy loss). We also see that the DGLV and the XDZR formalisms give similar results at RHIC energies, at the three rapidities under consideration. On the other hand, at LHC energy, the ASW and DGLV formalisms provide nearly identical results for energy

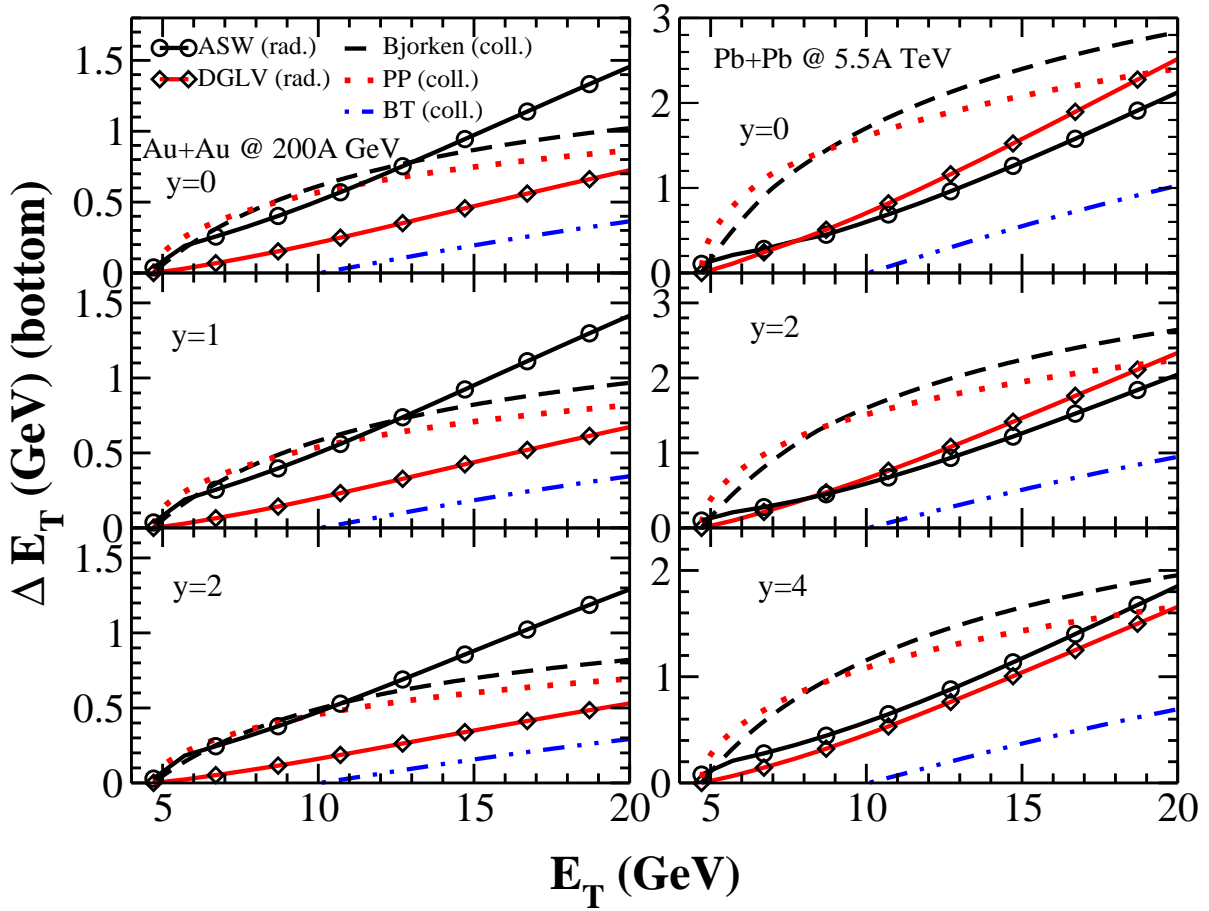


Figure 8. Same as Fig. 7 for a bottom quark

loss for charm quarks at $y=0$ and $y=2$, and the corresponding results at $y=4$ differ by about 10%. This, we feel, is due to a more complex dependence on the average path length in the ASW formulation.

The collisional energy loss for bottom quarks using the PP and the Bjorken's formulation are seen to be quite similar at RHIC and LHC energies for all rapidities under consideration. The BT formulation due to the neglect of the u-channel, gives a much smaller energy loss, both for RHIC and LHC energies and at all rapidities.

We have already mentioned that due to the numerical approximations used, the XDZR formulation is not valid for evaluation of the radiative energy loss for bottom quarks. The ASW and DGLV radiative energy loss formalisms show a more complex dependence on the mass and the average path length. The ASW formulation gives a larger energy loss at RHIC energy, though the results are again comparable at LHC energy at all rapidities. We note that while the collisional and radiative energy losses for bottom quarks at RHIC energy are comparable, the collisional energy loss dominates over the radiative energy loss in the E_T range under consideration at LHC energy. We

have confirmed that at higher E_T , the radiative energy loss again starts dominating.

This rich structure suggests that description of energy loss for one (quark) mass at one rapidity, and one energy may not be enough to identify the most reliable treatments, for this.

6. R_{AA} for heavy quarks

The nuclear modification factor R_{AA} for heavy quarks can be expressed as:

$$R_{AA}(b) = \frac{dN^{AA}/d^2p_T dy}{T_{AA}(b) d\sigma^{NN}/d^2p_T dy}, \quad (12)$$

where, as mentioned earlier, $T_{AA}(b)$ is the nuclear overlap function for impact parameter b , calculated using Glauber model. We get $T_{AA} \approx 280 \text{ fm}^{-2}$ for Au+Au collisions at RHIC and $\approx 290 \text{ fm}^{-2}$ for Pb+Pb collisions at LHC, for $b=0$ fm.

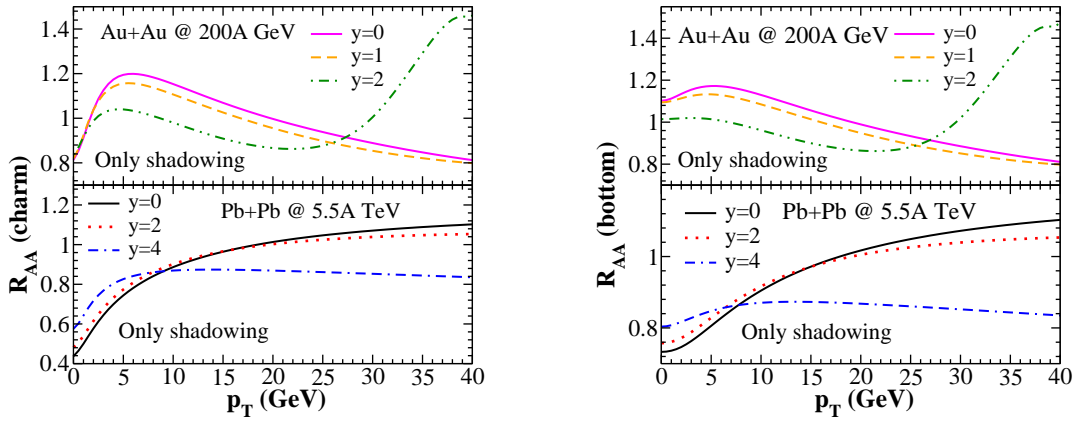


Figure 9. [Left panel] R_{AA} of charm quarks with only nuclear shadowing effect at more forward rapidities. [Right panel] Same as left panel for bottom quark.

As a first step, we give the results for R_{AA} with only nuclear shadowing effect for production of charm and bottom quarks at the rapidities considered earlier for RHIC and LHC energies (see Fig. 9). We see that the fairly large masses of the charm and bottom quarks, the kinematics, and the rich behaviour of the structure function with x and Q^2 , lends interesting features to R_{AA} .

We see that R_{AA} for charm and bottom quarks for $y=0$ and $y=1$ are quite similar at RHIC energy. Similarly, the results for $y=0$ and $y=2$ are only marginally different at LHC energy. The results at larger y are more strongly affected due to increased variation in the 'x' values (see Eq. 2) which contribute. In order to do a full justice to these interesting results, we now discuss them individually.

For charm quarks at RHIC energy, we see a suppression at lower p_T , an enhancement at intermediate p_T , and again a suppression at larger p_T , for $y=0$ and $y=1$, while for $y=2$, R_{AA} starts at about 0.8, goes up to a value slightly more than 1, then drops again to about 0.8 at $p_T \approx 20$ GeV, and rise again to beyond 1 at $p_T \approx 40$ GeV. Since the

energy loss of the charm quarks always rises with increasing p_T , this would introduce interesting features in R_{AA} after this is accounted for, unless of course the p_T spectrum for the quarks drops too rapidly. We shall come back to this point again. The increased energy at LHC then provides a larger suppression at low p_T for all the rapidities. In an interesting development, R_{AA} for $y=0$ and $y=2$ rises beyond 1 at $p_T \approx 20$ GeV, while it stays below 1 up to $p_T \approx 40$ GeV, for $y=4$.

The results for bottom quarks are even more interesting. Due to the large mass of the bottom quarks, at RHIC energy, the R_{AA} for lower p_T for $y=0$ and $y=1$ is already starts getting contributions from the region of x where anti-shadowing appears. Thus R_{AA} starts at a value which is more than 1 at lower p_T , goes up, up to $p_T \approx 5$ GeV and then drops again. For $y=2$ on the other hand, it starts at a value close to 1, drops by about 10 % at $p_T \approx 20$ GeV and rises again. At LHC energy, R_{AA} for bottom quarks for $y=0$ and $y=2$ starts at ≈ 30 % below 1 and then rises steadily to about 1.1 at $p_T \approx 40$ GeV. The results at $y=4$ remain close to 0.8, rising slightly at intermediate p_T .

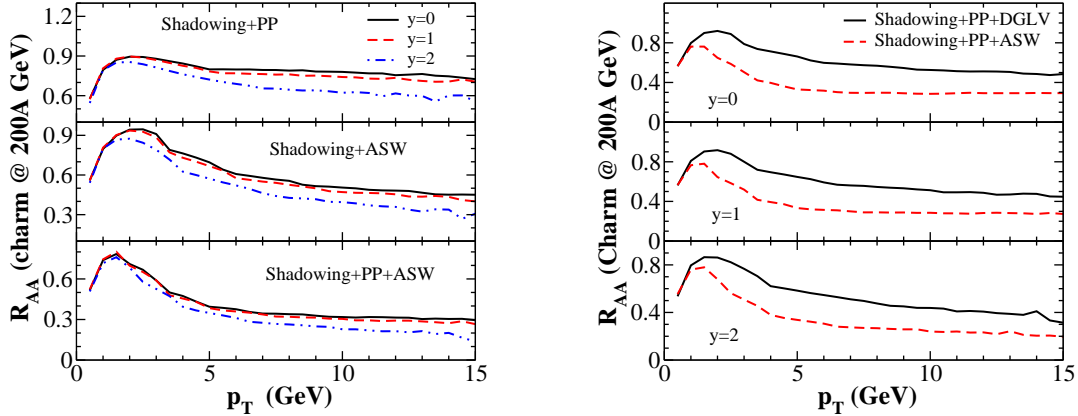


Figure 10. [Left panel] R_{AA} of charm quarks with the nuclear shadowing effect as well as the energy loss at more forward rapidities at RHIC energy. [Right panel] Comparison of the relative nuclear suppression using ASW and DGLV formalisms for charm quarks at different rapidities at RHIC energy.

Now let us discuss our results for R_{AA} with the additional inclusion of collisional and radiative energy losses. We shall restrict our consideration to inclusion of collisional energy loss using the PP formulation and the radiative energy loss using ASW or DGLV formulation.

Fig. 10 gives our findings for charm quarks at RHIC energy. We see that the outcome of shadowing and energy loss gives an interesting structure to R_{AA} , as expected. We see that the final R_{AA} starts at about 40 % below 1, goes up to about 0.8 at $p_T \approx 2$ GeV, and then drops to a value of ≈ 0.3 at $p_T \approx 15$ GeV. In an interesting development, we see that the combination of the shadowing and energy loss gives a marginally larger suppression at $y=2$ compared to $y=0$, even though the fractional energy loss is higher at smaller y (see Fig. 7). We have also given a comparison of R_{AA} by replacing the

ASW formulation for the radiative energy loss with the DGLV treatment, and see that the former gives a larger suppression at all y (see also Fig. 7). We recall that the single electrons produced from the semi-leptonic decay of charm mesons [26] show R_{AA} of about $0.2 \sim 0.3$ for $p_T > 2$ GeV, in a very encouraging agreement with these results. This will be published shortly.

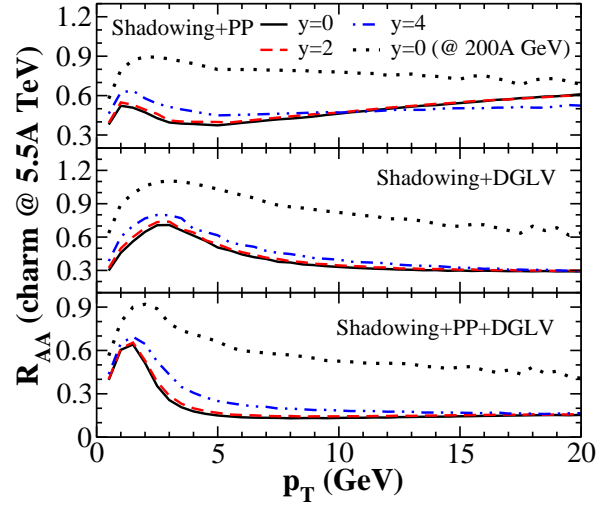


Figure 11. R_{AA} of charm quarks with the nuclear shadowing effect as well as the energy loss at more forward rapidities at LHC energy.

The results for the R_{AA} for charm quarks at LHC energy are shown in Fig. 11. We show the results only for the DGLV formulation for the radiative energy loss, as we have seen that it is quite similar to that for the ASW formalism for charm quarks at LHC energy. We see roughly similar behaviour, in that the R_{AA} starts from a lower value at $p_T \approx 0$ GeV, rises up to $p_T \approx 2$ GeV, and then drops to a level of about 0.2 at larger p_T . We also find a marginally larger suppression for $y = 0$ compared to that for $y = 4$. The results for $y = 0$ for RHIC energy are also given for a comparison which suggests a much larger suppression at LHC, as expected.

Next we discuss our findings for nuclear suppression for bottom quarks at RHIC energy (see Fig. 12). The shadowing and the large mass of the bottom quarks, with its consequences, gives an $R_{AA} \approx 0.8$ at $p_T \approx 0$ GeV, which goes up to about 1.2 at $p_T \approx 2$ GeV, and then drops to about $0.3 \sim 0.4$ at larger p_T . The shadowing results in a larger suppression for $y = 2$ compared to $y = 0$ (see Fig. 9), even though the energy loss is slightly lower for larger y (see Fig. 8). Results obtained by replacing the ASW formulation with the DGLV treatment show a smaller suppression, as for charm quarks (see Fig. 10).

Finally in Fig. 13 we have given our results for R_{AA} for bottom quarks at LHC energy for $y = 0, 2$ and 4 using shadowing, collisional energy loss and radiative energy loss using the DGLV treatment. The results using ASW treatment are expected to be quite similar as seen from Fig. 8. The results for $y = 0$ at RHIC energy are also given

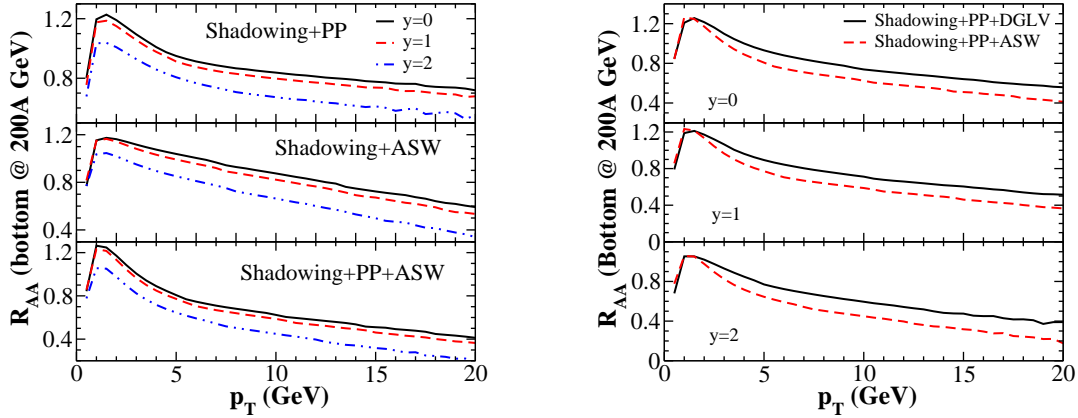


Figure 12. Same as Fig. 10 for bottom quarks.

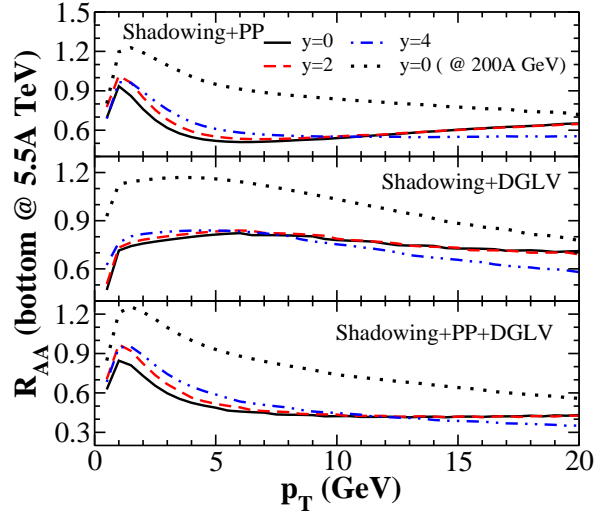


Figure 13. Same as Fig. 11 for bottom quarks.

for a ready reference. We again see a trend which is common to our results that R_{AA} starts at ≈ 0.6 at $p_T \approx 0$ GeV, goes up to ≈ 1.1 at $p_T \approx 2$ GeV, and then slowly drops to about 0.4 at $p_T \approx 20$ GeV. The overall effect of shadowing and energy loss is seen to lead to very similar values for R_{AA} from $y=0$ to $y=4$. Of course the R_{AA} for RHIC energy is about twice as large, showing a much reduced suppression.

7. Summary and Discussion

We have made a detailed study of charm and bottom production from initial fusion of partons in relativistic collision of heavy nuclei. As a first step we have checked the usefulness of lowest order pQCD to reproduce the NLO results for the p_T distribution of charm and bottom quarks in pp collisions. Next we have checked our predictions against experimental results of the charm and bottom quarks production from such collisions.

We have further obtained the average energy loss suffered by heavy quarks at several rapidities for RHIC and LHC energies due to collisions and gluon radiations. We have obtained the nuclear suppression factor R_{AA} by additionally incorporating nuclear shadowing for the cases under study. A rich picture of dependence of R_{AA} on y , p_T , incident energy, and the mass of the heavy quarks emerges. We have noted that our findings would support the suppression of single electrons seen at RHIC.

Before concluding, we discuss some of the short-comings of the present work. These initial calculations can be improved in several ways. We have used $(1+1)$ dimensional Bjorken hydrodynamics and assumed it to apply at all y . Since the heavy quarks will loose most of their energy at very early times, this may not be a serious short-coming. Still we are looking at the possibility of using a full fledged $(3+1)$ dimensional hydrodynamics calculations, also at $b \neq 0$. We are incorporating the single electron decay of the resulting D and B mesons, along with the results for their back to back correlation. We expect this to be rewarding, especially in conjunction with NLO results for pp collisions, as it may throw up an interesting detail about differences of NLO results and the results with energy loss. These will be published shortly.

Finally, we conclude that the description for energy loss for one quark mass at one rapidity for a particular incident energy may not be sufficient to identify the most reliable energy loss treatment for either collisional or radiative energy loss valid for all cases.

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Appendix

Appendix A. Collisional energy loss

A.1. Bjorken

Bjorken argued that the elastic energy loss by partons in the QGP is very similar to the energy loss due to ionization due to passage of charged particles in ordinary matter [58]. This treatment was adapted by Braaten and Thoma for heavy quarks [59]. The fractional energy loss suffered by the heavy quark due to collisions with the quarks and gluons given as:

$$\frac{dE}{dx} = \frac{8\pi\alpha_s^2 T^2}{3} \left(1 + \frac{N_f}{6}\right) \left[\frac{1}{v} - \frac{1-v^2}{2v^2} \log \frac{1+v}{1-v} \right] \log \frac{q_{\max}}{q_{\min}}, \quad (\text{A.1})$$

where v is the velocity of the heavy quark. As suggested by Braaten and Thoma we use the upper limit of the momentum transfer q_{\max} as $\sqrt{4TE}$ and the lower limit of the momentum transfer q_{\min} as $\sqrt{3m_g}$.

The thermal gluon mass m_g can be expressed as $m_g = \mu/\sqrt{2}$ where $\mu = \sqrt{4\pi\alpha_s T^2 \left(1 + \frac{N_f}{6}\right)}$ is the Debye screening mass.

A.2. Braaten and Thoma

Braaten and Thoma first developed a theoretical formalism to find the collisional energy loss of a muon propagating through a plasma of electrons, positrons and photons to leading order in QED [64]. This work was further extended by them to calculate the collisional energy loss of heavy quarks propagating through QGP [59]. The energy loss formulation is given in two energy regimes: The fractional collisional energy loss of heavy quarks with energy $E \ll M^2/T$ is

$$\frac{dE}{dx} = \frac{8\pi\alpha_s^2 T^2}{3} \left(1 + \frac{N_f}{6}\right) \left[\frac{1}{v} - \frac{1-v^2}{2v^2} \log \frac{1+v}{1-v} \right] \times \log \left(2^{\frac{N_f}{6+N_f}} B(v) \frac{ET}{m_g M} \right) \quad (\text{A.2})$$

The fractional collisional energy loss of heavy quarks with energy $E \gg M^2/T$ is

$$\frac{dE}{dx} = \frac{8\pi\alpha_s^2 T^2}{3} \left(1 + \frac{N_f}{6}\right) \log \left(2^{\frac{N_f}{6+N_f}} 0.92 \frac{\sqrt{ET}}{m_g} \right), \quad (\text{A.3})$$

where $B(v)$ is a smooth function of velocity having value in the range 0.6-0.7. Braaten and Thoma have shown the crossover energy between these energy regimes as $E_{\text{cross}} = 1.8 \times M^2/T$ for $N_f = 2$.

A.3. Peigne and Peshier

In BT formalism, it was assumed that the momentum exchange in the elastic scattering process is much less than the energy carried by the heavy quarks. Peigne and Peshier pointed out that this assumption is not reliable in the energy regime $E \gg M^2/T$, and corrected it in the QED case while calculating the collisional energy loss of a muon in QED plasma [65]. This work in QED is then used by them to derive the collisional energy loss suffered by heavy quarks while passing through QGP [60].

The fractional collisional energy loss suffered by heavy quarks as proposed by Peigne and Peshier is

$$\frac{dE}{dx} = \frac{4\pi\alpha_s^2 T^2}{3} \left[\left(1 + \frac{N_f}{6}\right) \log \frac{ET}{\mu^2} + \frac{2}{9} \log \frac{ET}{M^2} + c(N_f) \right] \quad (\text{A.4})$$

and $c(N_f) \approx 0.146 N_f + 0.05$.

Appendix B. Radiative energy loss

B.1. Djordjevic, Gyulassy, Levai, and Vitev

For massless quarks, Gyulassy, Levai and Vitev (GLV) calculated the induced radiation to arbitrary order in opacity χ^n ($\chi = L/\lambda$) of the plasma [66] where λ is the mean free path of the quark. In Djordjevic, Gyulassy, Levai, and Vitev (DGLV) formulation [61], the GLV method is generalized to estimate the first order induced radiative energy loss including the kinematic effect for heavy quarks. Wicks *et al.* [57] present a simplified form of the DGLV formalism for the average radiative energy loss of heavy quarks:

$$\Delta E = \frac{c_F \alpha_s E L}{\pi \lambda_g} \int_{\frac{m_g}{E+p}}^{1 - \frac{M}{E+p}} dx \int_0^\infty \frac{4 \mu^2 q^3 dq}{\left(\frac{4Ex}{L}\right)^2 + (q^2 + \beta^2)^2} (A \log B + C), \quad (\text{B.1})$$

where

$$\begin{aligned} \beta^2 &= m_g^2 (1 - x) + M^2 x^2, \\ \frac{1}{\lambda_g} &= \rho_g \sigma_{gg} + \rho_q \sigma_{qg}, \\ \sigma_{gg} &= \frac{9 \pi \alpha_s^2}{2 \mu^2}, \\ \sigma_{qg} &= \frac{4}{9} \sigma_{gg}, \\ \rho_g &= 16 T^3 \frac{1.202}{\pi^2}, \\ \rho_q &= 9 N_f T^3 \frac{1.202}{\pi^2}, \\ A &= \frac{2 \beta^2}{f_\beta^3} (\beta^2 + q^2), \\ B &= \frac{(\beta^2 + K) (\beta^2 Q_\mu^- + Q_\mu^+ Q_\mu^+ + Q_\mu^+ f_\beta)}{\beta^2 (\beta^2 (Q_\mu^- - K) - Q_\mu^- K + Q_\mu^+ Q_\mu^+ + f_\beta f_\mu)}, \\ C &= \frac{1}{2 q^2 f_\beta^2 f_\mu} [\beta^2 \mu^2 (2 q^2 - \mu^2) + \beta^2 (\beta^2 - \mu^2) K + Q_\mu^+ (\beta^4 - 2 q^2 Q_\mu^+) \\ &\quad + f_\mu (\beta^2 (-\beta^2 - 3 q^2 + \mu^2) + 2 q^2 Q_\mu^+) + 3 \beta^2 q^2 Q_k^-], \\ K &= (2 p x (1 - x))^2, \\ Q_\mu^\pm &= q^2 \pm \mu^2, \\ Q_k^\pm &= q^2 \pm K, \\ f_\beta &= f(\beta, Q_\mu^-, Q_\mu^+), \\ f_\mu &= f(\mu, Q_k^+, Q_k^-) \\ \text{and} \\ f(x, y, z) &= \sqrt{x^4 + 2 x^2 y + z^2}. \end{aligned} \quad (\text{B.2})$$

B.2. Armesto, Salgado, and Wiedemann

In Armesto, Salgado, and Wiedemann (ASW), formulation [62] path integral method for medium-induced gluon radiation is employed to calculate the radiative energy loss of heavy quarks. This formalism provides the analysis of the double differential medium-induced gluon distribution by the heavy quarks as a function of transverse momentum. The average radiative energy loss is

$$\begin{aligned} \Delta E = \frac{\alpha_s c_F}{\pi} (2 n_0 L) \int_0^E d\omega \int_0^{\frac{R}{2\gamma^2}} dk^2 \int_0^\infty dq^2 \times \\ \frac{(q^2 + \bar{M}^2) - \frac{1}{\gamma} \sin[\gamma(q^2 + \bar{M}^2)]}{(q^2 + \bar{M}^2)^2} \\ \times \frac{q^2}{q^2 + \bar{M}^2} \\ \times \frac{(k^2 + \bar{M}^2) + (k^2 - \bar{M}^2)(k^2 - q^2)}{(k^2 + \bar{M}^2) [(1 + k^2 + q^2)^2 - 4k^2 q^2]^{\frac{3}{2}}}. \quad (\text{B.3}) \end{aligned}$$

In the above equation the gluon energy, transverse momentum and heavy quark mass are expressed as dimensionless parameters. The rescaled dimensionless parameters:

$$\begin{aligned} \bar{M}^2 &\equiv \frac{1}{2} \left(\frac{M}{E} \right)^2 \frac{R}{\gamma^2}, \\ R = \omega_c L, \quad \omega_c &\equiv \frac{1}{2} \mu^2 L, \text{ and } \gamma \equiv \frac{\omega_c}{\omega}. \end{aligned} \quad (\text{B.4})$$

We use the parameter $n_0 L = 4$.

B.3. Xiang, Ding, Zhou, and Rohrich

In Xiang, Ding, Zhou, and Rohrich (XDZR) formulation [63], light cone-path integral method is used to calculate the gluon radiation from heavy quarks where an analytical expression is obtained for the heavy quark radiative energy loss.

The average radiative energy loss is

$$\begin{aligned} \Delta E = \frac{\alpha_s c_F}{4} \frac{L^2 \mu^2}{\lambda_g} \left[\log \frac{E}{\omega_{cr}} + \frac{m_g^2 L}{3 \pi \omega_{cr}} \left(1 - \frac{\omega_{cr}}{E} \log \frac{E^2}{2 \mu^2 L \omega_{cr}} + \log \frac{\omega_{cr}}{2 \mu^2 L} \right) \right. \\ \left. + \frac{M^2 L}{3 \pi E} \left(\frac{\pi^2}{6} - \frac{\omega_{cr}}{E} \log \frac{\omega_{cr}}{2 \mu^2 L} + \log \frac{E}{2 \mu^2 L} \right) \right], \quad (\text{B.5}) \end{aligned}$$

where $\omega_{cr} = 2.5$ GeV. This analytical expression is derived by using an expansion for Bessel function [63], which is valid only for not too large mass of the quarks. We use it only for charm quark.

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